Universal Dephasing Control During Quantum Computation

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Dephasing is a ubiquitous phenomenon that leads to the loss of coherence in quantum systems and the corruption of quantum information. We present a universal dynamical control approach to combat dephasing during all stages of quantum computation, namely, storage, single- and two-qubit operators. We show that (a) tailoring multi-frequency gate pulses to the dephasing dynamics can increase fidelity; (b) cross-dephasing, introduced by entanglement, can be eliminated by appropriate control fields; (c) counter-intuitively and contrary to previous schemes, one can increase the gate duration, while simultaneously increasing the total gate fidelity.

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Quantum computations, which promise to be faster than their classical analogs in a range of applications [1], can be performed via single-qubit and two-qubit operations only [2]. However, their experimental implementation has been proven to be difficult due to decoherence effects, which cause losses of quantum information [3, 4], particularly dephasing. Moreover, entanglement of qubits via two-qubit gates [5, 6], which is the cornerstone of quantum computation, results in faster loss of computational fidelity due to dephasing [7].

The problems of a single decohering qubit and its dynamical control have been thoroughly investigated, and recently extended to multipartite decoherence control [8]. Attempts have been made to combat dephasing during the storage stage, by applying sufficiently frequent, fast and strong pulses [9] or by introducing decoherence-free subspaces [10]. However, cross-dephasing due to entanglement, and the optimization of single- and two-qubit gate pulses so as to minimize dephasing [3, 8, 11] still need to be studied.

Here we present a universal dynamical-control approach aimed at suppressing dephasing during all stages of quantum information processing, namely, (i) information storage; (ii) manipulation by single-qubit gates, without entanglement, or (iii) by two-qubit gates that introduce entanglement. Our main results are to show that in order to reduce dephasing, it is advantageous to exert specific, addressable, dynamical control *on all the qubits at once*, whether or not they are manipulated by quantum gates. We show that the conventional approaches, whereby one tries to either reduce the gate duration or increase its coherence time, are not necessarily the best options in our control scheme. Instead, one can increase the gate duration and simultaneously reduce the effects of dephasing, resulting in higher gate fidelity. We introduce the multi-qubit system with its gates' implementation and arrive at a *general solution* to the problem of its dephasing. We then apply our (analytic) solution to two-qubit control and its fidelities, and use it in a detailed example of three-qubit computation.

Our system comprises N qubits, with ground and excited states $|g\rangle_j, |e\rangle_j$, respectively, and identical excitation energy $\hbar\omega_0$. Each qubit's excited state experiences random fluctuations, $\hbar\delta_j(t)$, thus introducing random dephasing. The total Hamiltonian is given by:

$$\hat{H} = \hat{H}^{(0)}(t) + \hat{H}^{(1)}(t) + \hat{H}^{(2)}(t) \tag{1}$$

$$\hat{H}^{(0)}(t) = \hbar\sum_{j=1}^{N}\left[\omega_0 + \delta_j(t)\right]|e\rangle_{jj}\langle e|\bigotimes_{k\neq j}\boldsymbol{I}_k \tag{2}$$

$$\hat{H}^{(1)}(t) = \hbar\sum_{j=1}^{N}\left(V_j^{(1)}(t)|e\rangle_{jj}\langle g| + H.c.\right)\bigotimes_{k\neq j}\boldsymbol{I}_k \tag{3}$$

$$\hat{H}^{(2)}(t) = \hbar\sum_{j=1}^{N}\sum_{k=j+1}^{N}\Big(V_{jk}^{(2)\Psi}(t)|ge\rangle_{jk}\langle eg| \tag{4}$$

$$+V_{jk}^{(2)\Phi}(t)|ee\rangle_{jk}\langle gg| + H.c.\Big)\bigotimes_{l\neq j,k}\boldsymbol{I}_l \tag{5}$$

where the superscript denotes the manipulation type (e.g., 1 and 2 for one- and two-qubit manipulation, respectively), and the subscript denotes the subject of manipulation. Here, $V_j^{(1)}(t)$ is the time-dependent single-qubit gate of the j-th qubit, $V_{jk}^{(2)\Psi,(2)\Phi}(t)$ are two possible time-dependent two-qubit gates, acting on qubits j and k, where the notation is derived from their diagonalization basis, i.e. the Bell-states basis, $|\Psi_\pm\rangle = 1/\sqrt{2}e^{-i\omega_0 t}(|eg\rangle \pm |ge\rangle)$, $|\Phi_\pm\rangle = 1/\sqrt{2}(e^{-i2\omega_0 t}|ee\rangle \pm |gg\rangle)$. Also, $\boldsymbol{I}$ is the identity matrix and $H.c.$ is Hermitian conjugate.

We treat the random dephasing, differently experienced by each qubit, as a stochastic Gaussian process with first and second ensemble-average-moments, $\overline{\delta_j(t)} = 0$, $\Phi_{jk}(t) = \overline{\delta_j(t)\delta_k(0)}$. We assume, for simplicity, that the driving fields of the single-qubit gates are resonant, with a time-dependent real envelope, i.e. $V_j^{(1)}(t) = \Omega_j^{(1)}(t)e^{-i\omega_0 t} + c.c.$, and the driving fields of the two-qubit gates are resonant on their transition, with a time-dependent real envelope, i.e. $V_{jk}^{(2)\Psi}(t) = \Omega_{jk}^{(2)\Psi}(t) + c.c$, and $V_{jk}^{(2)\Phi}(t) = \Omega_{jk}^{(2)\Phi}(t)e^{-i2\omega_0 t} + c.c.$. The rotating-wave approximation is used.

We consider three generic cases, namely, (a) only single-qubit gates are applied; (b) only two-qubit gates are applied on different pairs of qubits; and (c) single- and two-qubit gates are applied, where each qubit is either manipulated by a single- or a two-qubit gate, but never by both at once.

The single- ($q = 1$) and two- ($q = 2$) qubit cases can be solved by transforming to the interaction picture, and diagonalizing $\hat{H}^{(q)}(t)$. The diagonalizing basis for the entire Hamiltonian is then given by 2^N basis states, $|\Psi^{(q)}_{l_q}\rangle = \bigotimes_{j=1}^{N} |b_j^{l_q}\rangle_j$. Here, $l_1 = 0...2^N - 1$, $\{b_j^{l_1}\}$ is the binary representation of l_1, meaning $l_1 = b_1^{l_1} b_2^{l_1} ... b_N^{l_1}$, with $b_j^l = 0, 1$ corresponding to $|\pm\rangle_j = 1/\sqrt{2}(e^{-i\omega_0 t}|e\rangle_j \pm |g\rangle_j)$, respectively; and $l_2 = 0...2^N - 1$, $\{c_j^{l_2}\}$ is the quartary representation of l_2, meaning $l_2 = c_1^{l_2} c_2^{l_2} ... c_N^{l_2}$, with $c_j^{l_2} = 0, 1, 2, 3$ corresponding to $|\Psi_+, \Psi_-, \Phi_+, \Phi_-\rangle_{kk'}$, respectively. For the density matrix of the ensemble, $\overline{\rho}(t) = \overline{|\psi\rangle\langle\psi|}$, where $|\psi\rangle = \sum_{j=1}^{2^N} \beta_l(t)|\Psi_l^{(q)}\rangle$, the solution, to second order in $\delta_j(t)$, is then found to be:

$$\overline{\rho}(t) = \rho(0) - \int_0^t dt' \int_0^{t'} dt'' \overline{[\hat{W}^{(q)}(t'), [\hat{W}^{(q)}(t''), \rho(0)]]} \quad (6)$$

$$\hat{W}^{(q)}(t) = \sum_{l,m=1}^{2^N} w_{lm}^{(q)}(t) |\Psi_l^{(q)}\rangle\langle\Psi_m^{(q)}| \quad (7)$$

$$w_{lm}^{(q)}(t) = w_{ml}^{(q)*}(t) = \frac{1}{2} \begin{cases} \delta_j(t)\epsilon_j^{(1)}(t) & b_j^{l_1} = 0, b_j^{m_1} = 1 \\ & b_k^{l_1} = b_k^{m_1} \ \forall k \neq j \\ \delta_{j-}\epsilon_j^{(2)\Psi} & b_j^{l_2} = 0, b_j^{m_2} = 1 \\ & b_k^{l_2} = c_k^{m_2} \ \forall k \neq j \\ \delta_{j+}\epsilon_j^{(2)\Phi} & b_j^{l_2} = 2, b_j^{m_2} = 3 \\ & b_k^{l_2} = c_k^{m_2} \ \forall k \neq j \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where $\rho(0)$ is the initial density matrix, given in the diagonalized basis, $\delta_{j\pm}(t) = \delta_k(t) \pm \delta_{k'}(t)$. Here, $\epsilon_j^{(1)}(t) = e^{i\phi_j^{(1)}(t)}$, $\epsilon_j^{(2_\Psi, 2_\Phi)}(t) = e^{i\phi_{p_j}^{(2_\Psi, 2_\Phi)}(t)}$, where $\phi_j^{(1)}(t) = \int_0^t dt' \Omega_j^{(1)}(t')$ and $\phi_{p_j}^{(2_\Psi, 2_\Phi)}(t) = \int_0^t dt' \Omega_{p_j}^{(2_\Psi, 2_\Phi)}(t')$ are the accumulated phases.

This is the most general scheme of dephasing control analyzed thus far, in that it satisfies all the requirements for quantum computation, namely single- and two-qubit gates, applied simultaneously on different qubits. It is solvable by combining the results above with the solution given in the form of Eqs. (6), where the general interaction operator $\hat{W}(t)$ is a combination of Eqs. (8) with $q = 1, 2$. The three stages of quantum computation are defined by the restrictions on the overall phase accumulated by the state, due to the application of the gate fields at the end of each stage. During storage, the restrictions are $\phi_j^{(q)}(T) = 2\pi M_j \quad M_j = 0, \pm 1, \ldots$. To implement a Hadamard gate applied to the j-th qubit [12], the restrictions are $\phi_j^{(1)}(T) = \pi/4$ and storage regime for all the rest. To implement a SWAP gate between qubits k and k' [3], the restrictions are $\phi_{kk'}^{(2)\Psi}(T) = \pi/4$ and storage regime for all the rest.

To characterize the efficiency of the dephasing control schemes, we use fidelity, defined as $F(T) = \mathrm{Tr}(\rho_{\mathrm{target}}^{1/2} \overline{\rho}(T) \rho_{\mathrm{target}}^{1/2})$, where ρ_{target} is the target density matrix after the quantum computation, e.g. $\rho^{\mathrm{target}} = \rho(0)$ for the storage stage. The error of the gate operation is then $E(T) = 1 - F(T)$. However, since quantum computations require lack of knowledge of the initial qubits' state, we shall also use the average fidelity, $F_{avg}(T) = \langle F(T) \rangle$, where $\langle \cdots \rangle$ is the average over all possible initial pure states.

Armed with the general solutions and efficiency measures presented above, we now analyze in detail quantum computation by two qubits experiencing random dephasing. First, we apply single-qubit gates on each of the qubits. The average fidelity of this scheme is given by:

$$F_{avg}(T) = 1 - \tfrac{5}{12} \left(J_{11}^{(1)}(T) + J_{22}^{(1)}(T) \right) \quad (9)$$

$$J_{jk}^{(q)}(t) = \int_0^t dt' \int_0^{t'} dt'' \Phi_{jk}(t' - t'') \epsilon_j^{(q)}(t') \epsilon_k^{(q)*}(t'') \quad (10)$$

$$\mathrm{Re} J_{jk}^{(q)}(t) = \pi \int_{-\infty}^{\infty} d\omega G_{jk}(\omega) \epsilon_{j,t}^{(q)}(\omega) \epsilon_{k,t}^{(q)*}(\omega) \quad (11)$$

where $J_{jk}^{(q)}(t)$ is the modified dephasing function due to fields $\Omega_{j,k}^{(q)}$, $q = 1, 2_\Psi, 2_\Phi$. Here, $G_{jk}(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} dt \Phi_{jk}(t) e^{i\omega t}$ is the dephasing spectrum, and $\epsilon_{j,t}^{(q)}(\omega) = (2\pi)^{-1/2} \int_0^t dt' \epsilon_j^{(q)}(t') e^{i\omega t'}$ is the finite-time Fourier transform of the modulation (for a thorough analysis of the modified dephasing function, see Refs. [8]).

Equations (9)-(11) show the dependence of the fidelity on the spectral characteristics of the fields and the dephasing, and suggest how to tailor specific gate and control pulses: *reducing the spectral overlap* of the dephasing and modulation spectra [8]. Furthermore, they show that single-qubit gate fields do not cause cross-dephasing, since Eq. (9) depends only on single-qubit dephasing, $\Phi_{jj}(t)$. This comes about from the averaging over all initial qubits, where for each initial entangled state that "suffers" from cross-dephasing (e.g. triplet, $|\Phi_-\rangle$), there is another entangled state that "benefits" from cross-dephasing (e.g. the singlet, $|\Psi_-\rangle$). Equation (9) also shows that the modified dephasing function appears no matter what the accumulated phase, meaning that if one applies a gate field on one qubit, one can still benefit from applying a control field on the other, stored, qubit.

Next, we explore two-qubit gate operations. The average fidelity for this task is found to be:

$$F_{avg}(T) = 1 - \frac{5}{24} \sum_{j,k=1,2} \left(J_{jk}^{(2)\Phi}(T) + (-1)^{j+k} J_{jk}^{(2)\Psi}(T) \right) \quad (12)$$

Here we see that cross-dephasing does not cancel due to averaging, but has opposite signs for the different two-gate fields. Thus, for example, a SWAP gate may benefit from cross-dephasing. Furthermore, we see that applying both two-qubit gate fields can reduce dephasing, even if only one field is needed for the actual gate operation.

This means that applying a two-qubit storage gate field, with $\phi_{1,2}^{(2)\Phi}(T) = 2\pi M$, $M = 1, 2, \ldots$, along with, e.g., a SWAP gate, can reduce dephasing.

This novel approach, consisting in applying a storage gate field, concurrently with the actual gate field required for the logic operation, may result in longer gate durations, due to limitations on the gate fields themselves, such as minimal duration and maximal achievable peak-power. However, as seen from Eqs. (9), (12), this may still be beneficial if the reduction in the modulated dephasing due to the applied fields is greater than its increase due to longer gate duration.

We explore this approach in a specific, complex scenario of three qubits experiencing random dephasing. We accompany it by numerical results, where the dephasing is taken to be a set of random fluctuations with correlation-functions $\Phi_{jk}(t) = (\gamma/t_{c,jk})e^{-t/t_{c,jk}}\xi(|r_{jk}|)$, where γ is the asymptotic dephasing rate (without control fields $J(t \gg t_c) = \gamma t$), $t_{c,jk}$ is the corresponding correlation-time, and $\xi(|r_{jk}|)$ is the qubit-distance dependent cross-dephasing overlap function, with $\xi = 0(1)$ denoting no (maximal) cross-dephasing. We shall take $t_{c,jk} = t_c, \forall j, k$. For the gate fields we take realistic Gaussian pulses, with a restriction on their minimal duration and maximal peak-power, Fig. 1[inset]. The additional restrictions on the gate phases, e.g. $\phi_j^{(q)}(T) = 2\pi M_j$, leaves only one free parameter per field, namely M_j.

The initial state of the system is picked to be $|\psi(0)\rangle = |\uparrow\rangle_1 |e\rangle_2 |\downarrow\rangle_3$, where $|\uparrow(\downarrow)\rangle = \frac{1}{\sqrt{2}}(i|1\rangle \pm |0\rangle)$. Three single-qubit gates are applied, one per qubit, for time T_1. Then we store the first qubit, and apply gates on the other two, with the restrictions on the gate pulses being $\phi_1^{(1)}(T_1) = 2\pi M$; $\phi_2^{(1)}(T_1) = \pi/4$; and $\phi_3^{(1)}(T_1) = 7\pi/4$. The desired (target) state at the end of this stage is $\psi^{\text{target}}(T_1) = i|\uparrow\rangle_1 |\uparrow\rangle_2 |g\rangle_3$. In order to demonstrate the advantageous effects of complex dynamical gate sequences, and the benefits of longer gate durations while controlling the stored qubit, we compare our proposed approach (Fig. 1[inset]) and the conventional approach, whereby maximal peak-power and minimal duration Gaussian pulses are applied to achieve the required gates (Fig. 1, sequence 1). Our proposed approach requires longer gates, as the Gaussian pulses have a limited peak-power, and minimal duration. Comparing the proposed sequences 2 and 3 to the conventional sequence 1, demonstrates the trade-off between the beneficial effects of controlling the stored qubit and the detrimental effects of longer duration. For long correlation times, which are present in several experimental setups [13], additional increase in fidelity, shown in sequence 4, can be achieved by trains of short pulses, which reduce the dephasing due to higher frequencies in the dynamical control fields [8], in spite of its more than three-fold gate duration.

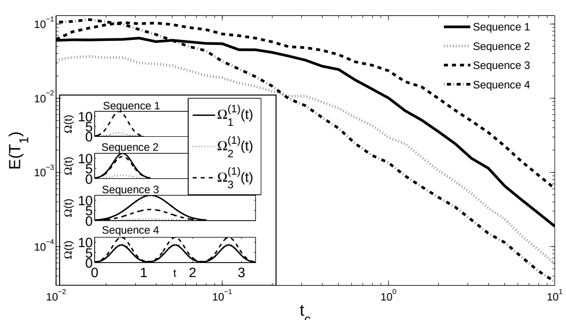

FIG. 1: Gate errors at the end of the first stage, $E(T_1)$ as a function of correlation time, t_c, for different dynamical gate fields [inset]. The gate fields parameters are $\phi_{1,2,3}^{(1)}(T_1) = \{0, \pi/4, 7\pi/4\}$, $\{2\pi, \pi/4, 7\pi/4\}$, $\{4\pi, \pi/4, 7\pi/4\}$ and $\{4\pi, 17\pi/4, 23\pi/4\}$ for sequences $1-4$, respectively. The gate durations are chosen such that the peak-power is the same for all sequences. Here $\gamma = 0.1$, and the results were obtained after averaging over 1000 realizations.

The next stage is to apply a two-qubit gate between the second and third qubits, and store the first one. We assume that $\rho(0) = |\psi^{\text{target}}(T_1)\rangle\langle\psi^{\text{target}}(T_1)|$, apply the $\Omega_{23}^{(2)\Psi}$ two-qubit gate on the second and third qubits, and use the other two gates to control the dephasing, resulting in gate pulse restrictions, $\phi_1^{(1)}(T_2) = 2\pi M$, $\phi_{23}^{(2)\Psi}(T_2) = 3\pi/2$, $\phi_{23}^{(2)\Phi}(T_2) = 2\pi M$. The desired state at the end of this stage is $|\psi^{\text{target}}(T_1 + T_2)\rangle = -i|\uparrow\rangle_1 |g\rangle_2 |-\rangle$. Figure 2 illustrates the effects of adding control fields concurrently with the desired SWAP gate. The application of the gate field reduces the modified dephasing function, $J_{23}^{(2)\Psi}$, making the other functions more dominant. Thus, one can observe the cross-dephasing overlap-dependent increase in gate error. However, introducing a control field for the second two-qubit gate eliminates this cross-dephasing dependence, resulting in a trade-off between gate-field duration increase and cross-dephasing decrease.

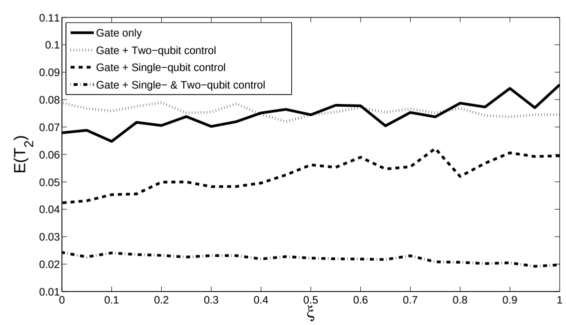

FIG. 2: Gate errors at the end of the second stage, $E(T_2)$ as a function of cross-dephasing overlap, ξ, for different dynamical gate fields. The gate fields parameters are $\{\phi_1^{(1)}, \phi_{23}^{(2)\Psi}, \phi_{23}^{(2)\Phi}\} = \{0, 3\pi/2, 0\}$ (solid), $\{0, 3\pi/2, 2\pi\}$ (dotted), $\{2\pi, 3\pi/2, 0\}$ (dashed) and $\{2\pi, 3\pi/2, 2\pi\}$ (dash-dot). The gate durations are chosen such that the peak-power is the same for all sequences. Here $\gamma = 0.1$, and the results were obtained after averaging over 1000 realizations.

Furthermore, introducing only control of the single-qubit field reduces the error, but leaves the cross-dephasing intact. Combining the two control schemes results in an even greater decrease in error, without cross-dephasing.

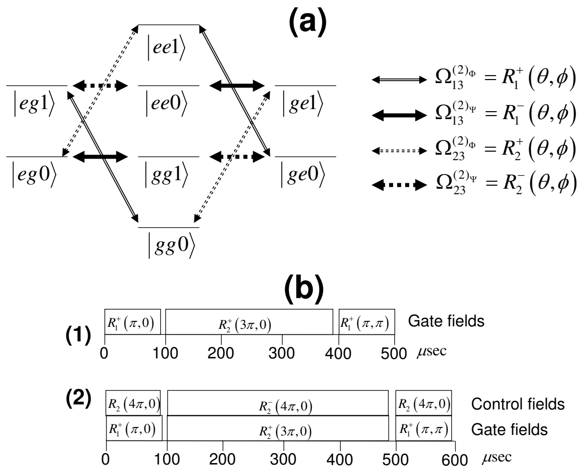

FIG. 3: (a) Schematic diagram of energy levels and two-qubit gate fields applied for two internal states of two ions $|g(e)\rangle$ and first two common vibrational levels $|0(1)\rangle$. (b) Conventional pulse sequence (1) and our proposed sequence (2). The pulse notation and parameters are taken from Ref. [5].

One implementation of these schemes may involve a string of ions in a linear trap [5, 14]. The qubits are encoded by two internal states of each ion ($|g(e)\rangle_j$), and are manipulated by individual-addressing laser beams. One introduces another qubit, encoded by the ground and first excited common vibrational levels ('bus-mode', $|0(1)\rangle_N$). The qubit gates are realized by applying laser pulses on the 'carrier' ($\Omega_j^{(1)}(t)$, $|g\rangle \leftrightarrow |e\rangle$), 'blue-sideband' ($\Omega_{jN}^{(2)\Phi}(t)$, $|g\rangle|0\rangle \leftrightarrow |e\rangle|1\rangle$) and 'red-sideband' ($\Omega_{jN}^{(2)\Psi}(t)$, $|g\rangle|1\rangle \leftrightarrow |e\rangle|0\rangle$) of the electronic quadrupole transition, Fig. 3(a). However, in a harmonic trap [5], the blue-(red-) sideband also couples to higher excitation levels, e.g. $|g\rangle|1\rangle \leftrightarrow |e\rangle|2\rangle$ ($|e\rangle|1\rangle \leftrightarrow |g\rangle|2\rangle$) and thus complicates the concurrent application of both two-qubit gates. This can be circumvented by imposing anharmonicity on the linear trap. Dephasing in the ion trap system can appear due to ambient magnetic field fluctuations that cause a Zeeman shift in the qubit levels. We have simulated a SWAP gate of two ions, using the first two common vibrational levels (assuming anharmonicity), with dephasing ($\gamma^{-1} = 1m$sec, $t_c = 300\mu$sec), Fig. 3(b). We compared the conventional pulse sequence Fig. 3(b.1), resulting in $F_{avg}(t = 500\mu\text{sec}) = 0.93$ and our proposed sequence Fig. 3(b.2), resulting in $F_{avg}(t = 600\mu\text{sec}) = 0.97$. This shows a considerable improvement in gate fidelity, despite its longer duration.

To conclude, we have formulated a universal protocol for dynamical dephasing control during all stages of quantum information processing, namely, storage, single- and two-qubit gate operations. It amounts to controlling all the qubits, whether they participate in the computation or not, and tailoring specific gate and control fields that optimally reduce the dephasing. This counter-intuitive protocol has a great advantage over others in that it increases the fidelity of the operation required, whether storage, manipulation or computation, despite the fact that it requires longer duration.

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